Scaling behavior of f_B with NRQCD [*]

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We investigate the scaling behavior of the B meson decay constant f_B and f_{B_s} at β=5.7, 5.9, 6.1, employing the NRQCD heavy quark action and the clover light quark action. Mixing effect from dimension-4 operator in the heavy-light axial-vector current is studied, and we find that the a dependence of f_B is significantly reduced. Our preliminary result for the decay constants in the quenched approximation is f_B=162$(^{+35}_{-18})$ MeV, f_{B_s}=190$(^{+40}_{-19})$ MeV, and f_{B_s}/f_B=1.18$(^{+6}_{-6})$.

1. Introduction

A recent development in the NRQCD study of heavy quarks on the lattice is the realization that the mixing of a dimension-4 operator with the axial-vector current, while nominally $O(\alpha_s a)$, has a significant effect in the value of the heavy-light decay constant[1,2]. An investigation of how this mixing effect affects the scaling behavior of the decay constant is an important issue.

In this work we study this problem, through simulations, with and without the operator mixing taken into account, at three values of β. A comparison is also made of the present NRQCD results with our previous calculation with the relativistic heavy quark action[3].

2. Method

We describe the light quark by the $O(a)$-improved SW clover action with one-loop corrected c_{sw} as in Ref. [3]. For heavy quark, we employ two types of the NRQCD action and op-

erator, one including all terms up to $O(1/M)$ and the other up to $O(1/M^2)$.

The $O(1/M)$ NRQCD action we use is

$$S=\sum_{t,\boldsymbol{x}}Q(t,\boldsymbol{x})\left[Q(t,\boldsymbol{x})-\left(1-\frac{aH_0}{2n}\right)^n\left(1-\frac{a\delta H}{2}\right)\right.$$
$$\left.\times U_4^\dagger\left(1-\frac{a\delta H}{2}\right)\left(1-\frac{aH_0}{2n}\right)^n Q(t-1,\boldsymbol{x})\right], \quad (1)$$

where Q is a two-component heavy quark field, $H_0=-\boldsymbol{\Delta}^{(2)}/[2M_0]$ and $\delta H=-g\boldsymbol{\sigma}\cdot\boldsymbol{B}/[2M_0]$. To the same order in $1/M$, the four-component Dirac field ψ_h is related to Q via FWT transformation,

$$\psi_h(x) = \left(1-\frac{\boldsymbol{\gamma}\cdot\boldsymbol{\Delta}^{(\pm)}}{2M_0}\right)\left(\begin{array}{c}Q(x)\\\chi^\dagger(x)\end{array}\right). \quad (2)$$

The mixing relation between the continuum axial-vector current and lattice counterparts, consistently expanded to $O(\alpha_s a)$ and $O(\alpha_s/M)$, is given by

$$A_4=\left[1+\alpha_s\rho_A^{(0)}\right]J^{(0)}+\alpha_s\rho_A^{(1)}J^{(1)}+\alpha_s\rho_A^{(2)}J^{(2)}, \quad (3)$$

where $J^{(0)}=\bar{\psi}_l\Gamma\psi_h$ with ψ_l the light quark field and $\Gamma=\gamma_5\gamma_4$, $J^{(1)}=-\bar{\psi}_l\Gamma\boldsymbol{\gamma}\cdot a\boldsymbol{\Delta}^{(\pm)}\psi_h$ and $J^{(2)}=\bar{\psi}_l$

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[*] Presented by K-I. Ishikawa.

Table 1
Parameters of simulation.

β	6.1	5.9	5.7
Vol.	$24^3 \times 64$	$16^3 \times 48$	$12^3 \times 32$
# of conf.	120	300	300
a^{-1}[GeV]	2.29	1.60	1.04

$\gamma \cdot a \overleftarrow{\boldsymbol{\Delta}}^{(\pm)} \Gamma \psi_h$. An important point observed in the first calculation of the one-loop coefficients $\rho_A^{(0,1,2)}$[1] is that the coefficient $\rho_A^{(2)}$ is not suppressed by $1/aM$ and remains as $O(1)$ for heavy quark, so that the mixing with the $J^{(2)}$ operator yields a large $O(\alpha_s a \Lambda_{\mathrm{QCD}})$ contribution. We have calculated the mixing coefficients for our $O(1/M)$ NRQCD action which is slightly different from that of Ref. [1].

3. Results on mixing effects

We carry out simulation at three values of β employing lattices and statistics as listed in Table 1. To set the lattice scale, we interpolate string tension data collected in Ref. [4] and set $\sqrt{\sigma}$=427 MeV.

Figure 1 shows our results for the quantity Φ_P $=(\alpha_s(M_P)/\alpha_s(M_B))^{2/\beta_0} f_P \sqrt{M_P}$ at β=5.9. We observe that the contribution of the mixing operators ($O(\alpha_s a)$), which is the difference between (○'s) and (●'s) in the figure, is as large as that of the multiplicative renormalization of the leading operator ($O(\alpha_s)$), which is the difference between (◇'s) and (○'s). This effect becomes more significant towards heavier quark mass due to a large value of $\rho_A^{(2)}$ and that of the matrix element of $J^{(2)}$, so that the slope of Φ_P becomes reduced with the inclusion of the mixing, as observed in Ref. [2]. We find this behavior to be more pronounced at $\beta = 5.7$.

In Fig. 2 we compare results for $O(1/M)$ NRQCD action with previous JLQCD results[3] obtained with the SW clover action for heavy quark, interpreted as a non-relativistic action within the Fermilab formalism[5], at β=6.1. Since the latter calculation does not include the effect of $O(\alpha_s a)$ mixing, we plot NRQCD results for the one-loop corrected leading operator. A good agreement of results for the two actions provides a check of viability of both the $1/M$ expansion
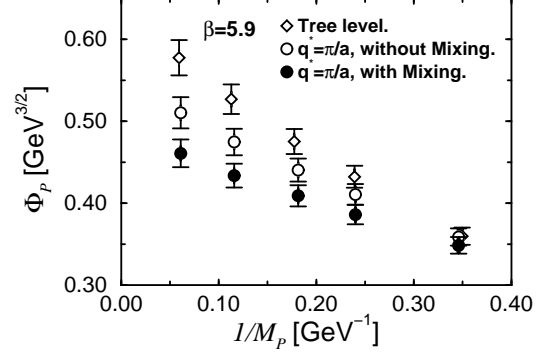

Figure 1. $1/M_{\mathrm{P}}$ dependence of Φ_{P} with the $O(1/M)$ NRQCD action at $\beta = 5.9$.
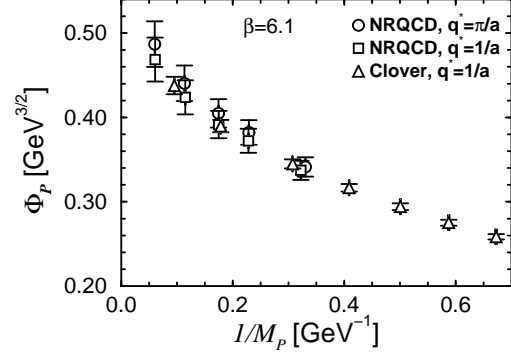

Figure 2. Φ_{P} with the $O(1/M)$ NRQCD action (this work) and with the SW clover action for heavy quark[3] at $\beta = 6.1$. Mixing effects are not included for both results.

approach of NRQCD and the Fermilab interpretation of the clover action for heavy quark.

Figure 3 presents the scaling behavior of f_B without (open symbols) and with (filled symbols) operator mixing, and for two choices of the momentum scale q^*=π/a and $1/a$ for the coupling constant. A large scatter of the values at $a^{-1} \approx 1$ GeV^{-1} (β=5.7) shows that one-loop estimates of renormalization factors are not reliable at such a large lattice spacing. This problem is substantially alleviated at β=5.9 and 6.1 ($0.4 \lesssim a \lesssim 0.6$ GeV^{-1}). In this region, the NRQCD result without including the operator mixing contribution has a large a dependence, which is sizably reduced with full inclusion of the mixing.

It is gratifying that the value of f_B in this range of a are reasonably consistent with the results
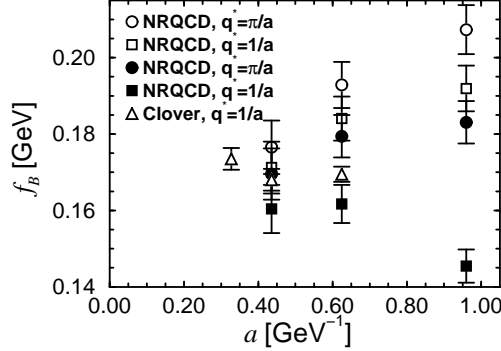
Figure 3. Scaling behavior of f_B with the $O(1/M)$ NRQCD action. Results with the SW clover action for heavy quark[3] are also plotted.

from the clover quark action ($\triangle$'s)[3] over the same range. Strictly speaking, such a comparison is to be made with the continuum extrapolated value of the latter. A mild scaling violation exhibited by the clover result suggests that the agreement would not be severely violated in such an extrapolation. Two points, however, have to be checked to consolidate the agreement: (i) the NRQCD values suffer from $O(\alpha_s/(aM)^2)$ errors toward smaller lattice spacing, whose magnitude in our range of a needs to be examined. (ii) The clover result does not incorporate effects of the $O(\alpha_s a)$ mixing, whose magnitude is yet unknown.

4. Results for decay constants

We estimate the physical value of the heavy-light decay constants from results at $\beta{=}6.1$ obtained with the $O(1/M)$ NRQCD action. Since the value of q^* is not known, we take the static result $q^*{=}2.18/a$ [6] as a guide, and calculate the central value from an average of results for $q^*{=}\pi/a$ and $1/a$. We then find that

$$f_B = 162(7)(5)(5)(11)(6)\binom{+31}{-8}\ \text{MeV}, \qquad (4)$$

$$f_{B_s} = 190(5)(5)(5)(13)(6)\binom{+39}{-9}\binom{+4}{-0}\ \text{MeV}. \qquad (5)$$

The first error is statistical including that from chiral extrapolation. Remaining are systematic errors arising from (i) the uncertainty of q^* estimated by dispersion of results for $q^*{=}\pi/a$ and $1/a$, (ii) $O(1/M^2)$ corrections estimated from comparison of results with the $O(1/M)$ and the

$O(1/M^2)$ calculations, (iii) $O(\alpha_s/(aM)^2)$ errors estimated by dividing $O(\alpha_s/(aM))$ contribution, which is derived from the result with static perturbative correction, by aM, (iv) scaling violation from comparison of the values at $\beta{=}6.1$ with those at $\beta{=}5.9$ and 5.7, and (v) uncertainty in a^{-1}, where the upper and lower errors correspond to the choice $a^{-1}{=}2.62$ GeV from charmonium 1s-1p splitting and 2.21 GeV from f_K as quoted in Ref. [7], respectively. For f_{B_s} the central value is obtained with κ_s for strange quark fixed by m_K, and the last error is estimated from the shift when κ_s is derived from m_ϕ. An $O(\alpha_s \Lambda_{\text{QCD}}/M)$ error coming from the action (1) is not included. A naïve estimate of this error gives $\sim$2% at $\beta{=}6.1$.

Some systematic errors cancel in the ratio

$$f_{B_s}/f_B = 1.18(3)(5)\binom{+2}{-0}, \qquad (6)$$

where the statistical error, scaling violation, and the uncertainty of κ_s, which remain, are given in this order.

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